Lesson 6 New Keynesian Model (1)

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- Lessons 4 and 5 introduce Classical Monetary Model which is benchmark of New Keynesian Model.
- Lessons 6 and 7 discuss New Keynesian Model by assuming monopolistical competitive market instead of perfect competitive market.
- By assuming monopolistical competitive market, firms can choose their prices to maximize their profit.
- The setting where firms choose their prices makes prices sticky. As we saw, data implies that prices are sticky and we introduce Calvo-pricing which makes prices sticky into the model.

- Although Calvo-pricing is not microfunded itself, it is widely used to analyze monetary policy because it makes model's prediction consistent with data easily.

14.1 The Model

- Similar to classical monetary model, households live infinitely.
- Firms act in monopolistical competitive market.
- Distortion stemming from monopolistical power is dissolved by taxation.
- Prices change following Calvo-pricing.

14.2 Households

- Similar to Eq.(3.1), households preference is given by:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{14.1}$$

Although C_t is consumption, different from classical monetary model, C_t is given by some index as follows:

$$C_t \equiv \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{14.2}$$

where $C(j)$ denotes amount of consumption on good j, $\varepsilon > 1$ denote the elasticity of substitution among goods.

- This index shows that there are goods on interval [0,1] infinitely and each good are different depending on the elasticity and sum of all of goods are 1.

- That is, each good are slightly different but substitutable. e.g., blue car, sky-blue car, white car….
- Eq.(14.2) is dubbed Dixit=Stiglitz aggregator or Constant Elasticity of Substitution aggregator because of constant elasticity of substitution.
- This assumption is useful to replicate monopolistical competitive market easily.
- The higher the ε, the higher the monopolistic power, and vice versa.
- ε is the inverse of the price elasticity.

- Households' budget constraint is given by:

$$\int_0^1 P_t(j)C_t(j)dj + Q_{t,t+1}\mathrm{E}_t\left(B_{t+1}\right) \le B_t + W_tN_t + TR_t \qquad (14.3)$$

The LHS is the sum of nominal amount of consumption and nominal pay-off of state contingent claim matured in next period while the RHS is the sum of nominal payoff of state contingent claim matured in current period, nominal wage and nominal lamp-sum transfer.

- The lamp-sum transfer consists of dividend from ownership of firms, tax and so fourth.
- Similar to classical monetary model, we impose the TVC $\lim_{k\to\infty}\mathrm{E}_0\left(B_kQ_{0,k}\right)=0$.

- Households have to think how many goods are purchased because there are a number of goods.
- The, households face maximization problem as follows:

$$\max_{C_t(i)} C_t$$

s.t.

$$\int_0^1 P_t(i)c_t(i)di \equiv Z_t$$

where Z_t denotes the nominal expenditure. This problem implies that households maximize the purchased amount of general good $C_t(i)$ by manipulating the amount subject to the budget constraint.

- The Lagrangean is given by:

$$L \equiv \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} + \lambda\left[Z_t - \int_0^1 P_t(j)C_t(j)dj\right]$$

- The FONC is given by:

$$\frac{\partial L}{\partial C_t(j)} = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}-1} C_t(j)^{\frac{\varepsilon-1}{\varepsilon}-1} - \lambda P_t(j) = 0$$

- This FONC can be rewritten as:

$$\lambda P_t(j) = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{1}{\varepsilon-1}} C_t(j)^{-\frac{1}{\varepsilon}}$$

$$= \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}\frac{1}{\varepsilon}} C_t(j)^{-\frac{1}{\varepsilon}}$$

$$= C_t^{\frac{1}{\varepsilon}} C_t(j)^{-\frac{1}{\varepsilon}} \qquad (14.4)$$

- Eq.(14.4) is aplicable for any goods $j \in [0,1]$ and we get:

$$\lambda P_t(j') = C_t^{\frac{1}{\varepsilon}} C_t(j')^{-\frac{1}{\varepsilon}} \qquad (14.5)$$

- Dividing Eq.(14.4) by Eq.(14.5) yields:

$$C_t(j) = C_t(j')\left[\frac{P_t(j)}{P_t(j')}\right]^{-\varepsilon}$$

- By plugging this into Eq.(14.2), we have:

$$C_t = \left\{\int_0^1 \left[C_t(j')\left(\frac{P_t(j)}{P_t(j')}\right)^{-\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}} dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \left[\int_0^1 C_t(j')^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{P_t(j)}{P_t(j')}\right)^{1-\varepsilon} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Thus, we get:

$$C_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj P_t(j')^{\varepsilon-1} C_t(j')^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} P_t(j')^{\varepsilon} C_t(j') \qquad (14.6)$$

- Let define:

$$P_t \equiv \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \qquad (14.7)$$

where P_t denotes the price index. Eq.(14.7) can be obtained by solving maximization problem with Eq.(14.2).

- By dividing the both sides of Eq.(14.7) by $-\varepsilon$, we get:

$$P_t^{-\varepsilon} = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Plugging this into Eq.(14.6) yields:

$$C_t = P_t^{-\varepsilon} P_t(j')^{\varepsilon} C_t(j')$$

- This equality is applicable for any goods *j*. Thus, we get the demand schedule for good *j* as follows:

$$C_t(i) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t \qquad (14.8)$$

- Eq.(14.8) implies that the relatively higher the price of good *j*, the relatively lower the demand for good *j*, and vice versa.

- Now, we consider the first term on the LHS in Eq.(14.3).
- Plugging Eq.(14.8) into the first term on the LHS in Eq.(14.3) yields:

$$\int_0^1 P_t(j)C_t(j)di = \int_0^1 P_t(j)^{1-\varepsilon} di P_t^{\varepsilon} C_t$$
$$= \left[\int_0^1 P_t(j)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}1-\varepsilon} P_t^{\varepsilon} C_t$$
$$= P_t^{1-\varepsilon} P_t^{\varepsilon} C_t$$
$$= P_t C_t$$

- Plugging this into Eq.(14.3) yields:

$$P_t C_t + Q_{t,t+1} \mathrm{E}_t(B_{t+1}) \leq B_t + W_t N_t + TR_t \qquad (14.9)$$

which is a budget constraint and is definitely same as it in classical monetary model.

- Assuming monopolistical competitive market makes the model difficult to handle because of a number of goods at glance. However, as long as the CES aggregator is adopted, handling the model is not difficult but easy.
- In fact, the FONCs for households are same as those in classical monetary model.
- Similar to classical monetary model, households maximize Eq.(14.1) subject to Eq.(14.9).

- The optimality conditions for households are given by:

$$\beta \mathrm{E}_t\left(\frac{U_{C_{t+1}}}{U_{C_t}} \frac{P_t}{P_{t+1}}\right) = \frac{1}{1+i_t}$$
$$-\frac{U_{N_t}}{U_{C_t}} = \frac{W_t}{P_t}$$

- Similar to classical monetary model, $Q_{t,t+1} = 1/(1+i_t)$ is applied because stochastic discount factor equals to discount rate on discount bonds.
- Similar to classical monetary model, we assume Additively Separable, namely $U_{C_t N_t} = 0$ is applied.

- For simplicity without loss of generality, we assume $U(C_t, N_t) = \ln C_t - 1/2 N_t^2$ similar to classical monetary model.
- The, households optimality conditions are:

$$\beta \mathrm{E}_t\left(\frac{P_t C_t}{P_{t+1} C_{t+1}}\right) = \frac{1}{1+i_t} \qquad (14.10)$$
$$N_t C_t = \frac{W_t}{P_t} \qquad (14.11)$$

- Eqs.(14.10) and (14.11) are definitely same as Eqs.(3.12) and (3.13), which are optimality conditions in classical monetary model.

- Similar to classical monetary model, we assume the steady state in which the inflation is zero.
- By log-linearizing Eq.(14.10) around the steady state, we have:

$$c_t = \mathrm{E}_t(c_{t+1}) - \hat{i}_t + \mathrm{E}_t(\pi_{t+1}) \qquad (14.12)$$

which is same as Eq.(3.20).

- Although we assume monopolistical competitive market instead of perfect competitive market, aggregated households behavior is not different.
- That is, the building block of households is same as it in classical monetary model.

- Similar to classical monetary model, we assume the real money demand function as follows:

$$\frac{M_t}{P_t} = Y_t i_t^{-\eta}$$

- By log-lineraizing this, we have:

$$m_t - p_t = y_t - \eta \hat{i}_t \quad (14.13)$$

14.3 Firms

- Good j is produced by firm $j \in [0,1]$ having production function as follows:

$$Y_t(j) = A_t N_t(j) \quad (14.14)$$

where $Y_t(j)$ denotes the output of good j and $N_t(j)$ denotes the employment toprodce good j. Eq.(14.14) implies that each firm j has same productivity A_t.

- The equilibrium condition in the labor market is given by:

$$\int_0^1 N_t(j) \equiv N_t$$

- Different from classical monetary model, we assume constant return on scale for simplicity.
- The assumption is (almost) analogous to assuming $\alpha = 0$ in classical monetary model.

14.3.1 Dynamics on Prices

- Firms act in monopolistical competitive market and choose their prices to maximize their profit.
- Further, we assume Calvo-pricing. Under this setting, just fraction θ firms can choose optimal prices to maximize their profit and fraction $1-\theta$ firms cannot choose optimal price and their prices remains in previous period.
- In this setting, $(1-\theta)^{-1}$ corresponds to the duration of price revision.

Fig. 13-1: The Duration of Price Revision and the Price Stickiness

	The Duration of Price Revision		Price Stickiness
	Month	Quarter	
	$3 \times 1/(1-\theta)$	$1/(1-\theta)$	θ
Taylor (1999)	12	4	0.75
Bils and Klenow (2004)	4~6	1.33~2	0.25~0.5
Nakamura and Steinsson (2006)	8~11	2.67~3.67	0.63~0.73

- Lesson 4 introduces Taylor (1999)、Bils and Klenow (2004)、Nakamura and Steinsson (2006) who analyze the duration empilically.
- We can know the price stickiness from the price stickiness θ .

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- For example, Taylor (1999) shows that the duration is approximately 12 months which corresponds to the price stickiness is 0.75.
- That is, 25% of firms choose optimal prices while 75% firms' prices remain.

- Calvo-pricing's transitory equation is given by:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\tilde{P}_t^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

- Log-linearizing this around the steady state where $\Pi_t \equiv P_t / P_{t-1} = 1$ and $\tilde{P}_t / P_t = 1$ are applied yields:

$$p_t = \theta p_{t-1} + (1-\theta)\tilde{p}_t \quad (14.15)$$

- Eq.(14.15) shows that the price is sum of revised and non-revised prices. When all firms choose optimal price, namely, $\theta = 0$, Eq. (14.15) boilds down to $p_t = \tilde{p}_t$. That is, the price corresponds to optimal price.

- Subtracting p_t _1 from the both sides of Eq.(14.15) yields:

$$\pi_t = (1-\theta)(\tilde{p}_t - p_{t-1}) \quad (14.16)$$

- Eq.(14.16) shows that inflation stems from the difference between optimal price and previous price which is an average of prices chosen by firms in previous period.
- That is, the higher the optimal price relatively, the higher the inflation, and vice versa.
- What is behind firms' decision on prices?

14.3.2 Optimal Price Setting

- To maximize their profit, firms choose $\tilde{P}_t$ in each period.
- The optimization problem facing firms is given by:

$$\max_{\tilde{P}_t} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} \left[\tilde{P}_t \tilde{Y}_{t+k|t} - \Psi\left(\tilde{Y}_{t+k|t}\right) \right] \right\}$$

where $\tilde{Y}_{t+k|t} \equiv \left(\tilde{P}_t / P_{t+k}\right)^{-\varepsilon} C_{t+k}$ denotes a demand for goods under nominal rigidity.

- The higher the optimal price $\tilde{P}_t$, the lower the demand for goods, and vice versa.
- $\Psi\left(\tilde{Y}_{t+k|t}\right)$ is the (nominal) cost function.

- Firms' profit is discounted by not only the stochastic discount factor $Q_{t,t}$ but also the price stickiness θ .
- This implies that firms choose their prices with calculating on giving up to choosing optimal price.
- This can be understood by regarding θ as a probability of that firms cannot their prices.

- The FONC is given by:

$$\sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left[Q_{t,t+k} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}^n \right) \tilde{Y}_{t+k|t} \right] = 0 \quad (14.17)$$

where MC_t^n is the nominal marginal cost.

- $\varepsilon / (\varepsilon - 1)$ is constant mark up over time.
- Eq.(14.17) implies that firms choose their prices which equal to net present value of nominal marginal cost with constant mark up.

- When there is no price stickiness, namely, $\vartheta = 0$, Eq.(14.17) boilds down to:

$$\tilde{P}_t = \frac{\varepsilon}{\varepsilon-1} MC_t^n$$

By ignoring constant mark up, we see that prices equal to the nominal marginal cost, similar to classical monetary model.

- New Keynesian model assumes monopolistical competitive market. Because of this, firms choose prices equal to the nominal marginal cost with constant markup when there is no price stickiness. Eq.(14.17) implies this fact.

- By log-linearizing Eq.(14.17), we get:

$$\tilde{p}_t - p_{t-1} = (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(mc_{t+k} \right) + \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(\pi_{t+k} \right) \quad (14.18)$$

- Rearranging Eq.(14.18) yields:

$$\begin{aligned} \tilde{p}_t &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(mc_{t+k} + p_{t+k} \right) \\ &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(mc_{t+k}^n \right) \quad (14.19) \end{aligned}$$

- Eq.(14.19) shows that firms choose prices equal to net present value of the nominal marginal cost.

Proof of Eq.(14.17)

- Firms' optimization problem can be rewritten as:

$$\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t\left\{Q_{t,t+k}\left[\tilde{P}_t\tilde{Y}_{t+k|t}-\Psi\left(\tilde{Y}_{t+k|t}\right)\right]\right\}$$

$$=Q_{t,t}\left\{\tilde{P}_t\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_t-\Psi\left[\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_t\right]\right\}$$

$$+\theta\mathrm{E}_t\left\{Q_{t,t+1}\left(\tilde{P}_t\left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{-\varepsilon}C_{t+1}-\Psi\left[\left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{-\varepsilon}C_{t+1}\right]\right)\right\}$$

$$+\theta^2\mathrm{E}_t\left\{Q_{t,t+2}\left(\tilde{P}_t\left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{-\varepsilon}C_{t+2}-\Psi\left[\left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{-\varepsilon}C_{t+2}\right]\right)\right\}+\cdots$$

$$=Q_{t,t}\left[\tilde{P}_t^{1-\varepsilon}P_t^{\varepsilon}C_t-\Psi\left(\tilde{P}_t^{-\varepsilon}P_t^{\varepsilon}C_t\right)\right]$$

$$+\theta\mathrm{E}_t\left\{Q_{t,t+1}\left[\tilde{P}_t^{1-\varepsilon}P_{t+1}^{\varepsilon}C_{t+1}-\Psi\left(\tilde{P}_t^{-\varepsilon}P_{t+1}^{\varepsilon}C_t\right)\right]\right\}$$

$$+\theta^2\mathrm{E}_t\left\{Q_{t,t+2}\left[\tilde{P}_t^{1-\varepsilon}P_{t+2}^{\varepsilon}C_{t+2}-\Psi\left(\tilde{P}_t^{-\varepsilon}P_{t+2}^{\varepsilon}C_t\right)\right]\right\}+\cdots$$

- The FONC of this is given by:

$$Q_{t,t}\left[(1-\varepsilon)\tilde{P}_t^{-\varepsilon}P_t^{\varepsilon}C_t-\Psi'\left(\tilde{Y}_{t|t}\right)(-\varepsilon)\tilde{P}_t^{-\varepsilon-1}P_t^{\varepsilon}C_t\right]$$

$$+\theta\mathrm{E}_t\left\{Q_{t,t+1}\left[(1-\varepsilon)\tilde{P}_t^{-\varepsilon}P_{t+1}^{\varepsilon}C_{t+1}-\Psi'\left(\tilde{Y}_{t+1|t}\right)(-\varepsilon)\tilde{P}_t^{-\varepsilon-1}P_{t+1}^{\varepsilon}C_{t+1}\right]\right\}$$

$$+\theta^2\mathrm{E}_t\left\{Q_{t,t+2}\left[(1-\varepsilon)\tilde{P}_t^{-\varepsilon}P_{t+2}^{\varepsilon}C_{t+2}-\Psi'\left(\tilde{Y}_{t+2|t}\right)(-\varepsilon)\tilde{P}_t^{-\varepsilon}P_{t+2}^{\varepsilon}C_{t+2}\right]\right\}+\cdots$$

$$=0$$

- Multiplying $\tilde{P}_t\cdot\dfrac{1}{1-\varepsilon}$ on the both sides of this yields:

$$Q_{t,t}\left[\tilde{P}_t\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_t-\frac{\varepsilon}{\varepsilon-1}\Psi'\left(\tilde{Y}_{t|t}\right)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_t\right]$$

$$+\theta\mathrm{E}_t\left\{Q_{t,t+1}\left[\tilde{P}_t\left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{-\varepsilon}C_{t+1}-\frac{\varepsilon}{\varepsilon-1}\Psi'\left(\tilde{Y}_{t+1|t}\right)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_{t+1}\right]\right\}$$

$$+\theta^2\mathrm{E}_t\left\{Q_{t,t+2}\left[\tilde{P}_t\left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{-\varepsilon}C_{t+2}-\frac{\varepsilon}{\varepsilon-1}\Psi'\left(\tilde{Y}_{t+2|t}\right)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_{t+2}\right]\right\}+\cdots$$

$$=0$$

- This can be rewritten as:

$$Q_{t,t}\left\{\left[\tilde{P}_t-\frac{\varepsilon}{\varepsilon-1}\Psi'\left(\tilde{Y}_{t|t}\right)\right]\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_t\right\}$$

$$+\theta\mathrm{E}_t\left\{Q_{t,t+1}\left[\left(\tilde{P}_t-\frac{\varepsilon}{\varepsilon-1}\Psi'\left(\tilde{Y}_{t+1|t}\right)\right)\left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{-\varepsilon}C_{t+1}\right]\right\}$$

$$+\theta^2\mathrm{E}_t\left\{Q_{t,t+2}\left[\left(\tilde{P}_t-\frac{\varepsilon}{\varepsilon-1}\Psi'\left(\tilde{Y}_{t+2|t}\right)\right)\left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{-\varepsilon}C_{t+2}\right]\right\}+\cdots$$

$$=0$$

- Because $\Psi'\left(\tilde{Y}_{t+k|t}\right)$ is the nominal marginal cost, we plug $MC_{t+k}^n\equiv\Psi'\left(\tilde{Y}_{t+k|t}\right)$ into this and we get:

$$Q_{t,t}\left[\left(\tilde{P}_t-\frac{\varepsilon}{\varepsilon-1}MC_t^n\right)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\varepsilon}C_t\right]$$

$$+\theta\mathrm{E}_t\left\{Q_{t,t+1}\left[\left(\tilde{P}_t-\frac{\varepsilon}{\varepsilon-1}MC_{t+1}^n\right)\left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{-\varepsilon}C_{t+1}\right]\right\}$$

$$+\theta^2\mathrm{E}_t\left\{Q_{t,t+2}\left[\left(\tilde{P}_t-\frac{\varepsilon}{\varepsilon-1}MC_{t+2}^n\right)\left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{-\varepsilon}C_{t+2}\right]\right\}+\cdots$$

$$=0$$

- By using summation symbol, this can be rewritten as

$$\sum_{k=0}^{\infty}\theta^k\mathrm{E}_t\left\{Q_{t,t+k}\left[\left(\tilde{P}_t-\frac{\varepsilon}{\varepsilon-1}MC_{t+k}^n\right)\left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\varepsilon}C_{t+k}\right]\right\}=0$$

which equals to Eq.(14.17) obviously.

QED

Proof of Eq.(14.18)

- The stochastic discount factor from period t to period $t+k$ is discount rate of the bond purchased in period t and matured in period $t+k$. Thus, we have:

$$Q_{t,t+k} = \beta^k \left(\frac{P_t C_t}{P_{t+k} C_{t+k}} \right) \qquad (14.20)$$

- Pluging Eq.(14.20) into Eq.(14.17) yields:

$$\sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left[\frac{P_t C_t}{P_{t+k} C_{t+k}} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k}^n \right) \tilde{Y}_{t+k|t} \right]$$
$$= \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k}^n \right) \tilde{Y}_{t+k|t} + \theta\beta \mathrm{E}_t \left[\frac{P_t C_t}{P_{t+1} C_{t+1}} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+1}^n \right) \tilde{Y}_{t+1|t} \right]$$
$$+ (\theta\beta)^2 \mathrm{E}_t \left[\frac{P_t C_t}{P_{t+2} C_{t+2}} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+2}^n \right) \tilde{Y}_{t+2|t} \right] + \cdots$$
$$= 0$$

- Dividing the both sides of this equality by $P_t C_t$ yields:

$$\left(P_t C_t \right)^{-1} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k}^n \right) \tilde{Y}_{t+k|t}$$
$$+ \theta\beta \mathrm{E}_t \left[\left(P_{t+1} C_{t+1} \right)^{-1} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+1}^n \right) \tilde{Y}_{t+1|t} \right]$$
$$+ (\theta\beta)^2 \mathrm{E}_t \left[\left(P_{t+2} C_{t+2} \right)^{-1} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+2}^n \right) \tilde{Y}_{t+2|t} \right] + \cdots$$
$$= \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left[\left(P_{t+k} C_{t+k} \right)^{-1} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k}^n \right) \tilde{Y}_{t+k|t} \right]$$
$$= 0$$

Here, $(P_t C_t)^{-1}$ can be regarded as marginal utility of nominal consumption.

- Multiplying P_{t-1}/P_{t-1} and dividing the nominal marginal cost by the price . This yields:

$$C_t^{-1} \tilde{Y}_{t+k|t} \frac{P_{t-1}}{P_t} \left(\frac{\tilde{P}_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_t}{P_{t-1}} MC_t \right)$$
$$+ \theta\beta \mathrm{E}_t \left[C_{t+1}^{-1} \tilde{Y}_{t+1|t} \frac{P_{t-1}}{P_{t+1}} \left(\frac{\tilde{P}_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{t+1}}{P_{t-1}} MC_{t+1} \right) \right]$$
$$+ (\theta\beta)^2 \mathrm{E}_t \left[C_{t+2}^{-1} \tilde{Y}_{t+2|t} \frac{P_{t-1}}{P_{t+2}} \left(\frac{\tilde{P}_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{t+2}}{P_{t-1}} MC_{t+2} \right) \right] + \cdots$$
$$= 0$$

- Then:

$$\sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left[C_{t+k}^{-1} \tilde{Y}_{t+k|t} \frac{P_{t-1}}{P_{t+k}} \left(\frac{\tilde{P}_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} \Pi_{t-1,t+k} MC_{t+1} \right) \right] = 0$$

where $MC_t \equiv MC_t^n / P_t$ denotes the real marginal cost with $\Pi_{t-1,t+k} \equiv P_{t+k} / P_{t-1}$.

- This can be rewritten as:

$$C_t^{-1} \tilde{Y}_{t|t} \frac{P_{t-1}}{P_t} \left(\frac{\tilde{P}_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_t}{P_{t-1}} MC_t \right)$$
$$+ \theta\beta \mathrm{E}_t \left[C_{t+1}^{-1} \tilde{Y}_{t+1|t} \frac{P_t}{P_{t+1}} \frac{P_{t-1}}{P_t} \left(\frac{\tilde{P}_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_t}{P_{t-1}} \frac{P_{t+1}}{P_t} MC_{t+1} \right) \right]$$
$$+ (\theta\beta)^2 \mathrm{E}_t \left[C_{t+2}^{-1} \tilde{Y}_{t+2|t} \frac{P_{t+1}}{P_{t+2}} \frac{P_t}{P_{t+1}} \frac{P_{t-1}}{P_t} \left(\frac{\tilde{P}_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_t}{P_{t-1}} \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_{t+1}} MC_{t+2} \right) \right] + \cdots$$
$$= 0$$

- Further:

$$
\begin{aligned}
&C_t^{-1}\tilde{Y}_{t|t}\Pi_t^{-1}\left(\tilde{X}_t-\frac{\varepsilon}{\varepsilon-1}\Pi_t MC_t\right)\\
&+\theta\beta \mathrm{E}_t\left[C_{t+1}^{-1}\tilde{Y}_{t+1|t}\Pi_{t+1}^{-1}\Pi_t^{-1}\left(\tilde{X}_t-\frac{\varepsilon}{\varepsilon-1}\Pi_t\Pi_{t+1}MC_{t+1}\right)\right]\\
&+(\theta\beta)^2\mathrm{E}_t\left[C_{t+2}^{-1}\tilde{Y}_{t+2|t}\Pi_{t+2}^{-1}\Pi_{t+1}^{-1}\Pi_t^{-1}\left(\tilde{X}_t-\frac{\varepsilon}{\varepsilon-1}\Pi_t\Pi_{t+1}\Pi_{t+2}MC_{t+2}\right)\right]+\cdots\\
&=0
\end{aligned}
$$

with $\tilde{X}_t\equiv\tilde{P}_t/P_{t-1}$.

- Further:

$$
\begin{aligned}
&C_t^{-1}\tilde{Y}_{t|t}\Pi_t^{-1}\tilde{X}_t-\frac{\varepsilon}{\varepsilon-1}C_t^{-1}\tilde{Y}_{t+k|t}MC_t\\
&+\theta\beta \mathrm{E}_t\left[C_{t+1}^{-1}\tilde{Y}_{t+1|t}\Pi_{t+1}^{-1}\Pi_t^{-1}\tilde{X}_t-\frac{\varepsilon}{\varepsilon-1}C_{t+1}^{-1}\tilde{Y}_{t+1|t}MC_{t+1}\right]\\
&+(\theta\beta)^2\mathrm{E}_t\left[C_{t+2}^{-1}\tilde{Y}_{t+2|t}\Pi_{t+2}^{-1}\Pi_{t+1}^{-1}\Pi_t^{-1}\tilde{X}_t-\frac{\varepsilon}{\varepsilon-1}C_{t+2}^{-1}\tilde{Y}_{t+2|t}MC_{t+2}\right]+\cdots\\
&=0
\end{aligned}
$$

- By moving the terms related to the marginal cost to the RHS yields:

$$
\begin{aligned}
&C_t^{-1}\tilde{Y}_{t|t}\Pi_t^{-1}\tilde{X}_t\\
&+\theta\beta \mathrm{E}_t\left(C_{t+1}^{-1}\tilde{Y}_{t+1|t}\Pi_{t+1}^{-1}\Pi_t^{-1}\tilde{X}_t\right)\\
&+(\theta\beta)^2\mathrm{E}_t\left(C_{t+2}^{-1}\tilde{Y}_{t+2|t}\Pi_{t+2}^{-1}\Pi_{t+1}^{-1}\Pi_t^{-1}\tilde{X}_t\right)+\cdots\\
&=\frac{\varepsilon}{\varepsilon-1}\left\{\begin{array}{l}C_t^{-1}\tilde{Y}_{t+k|t}MC_t+\theta\beta\mathrm{E}_t\left(C_{t+1}^{-1}\tilde{Y}_{t+1|t}MC_{t+1}\right)\\+(\theta\beta)^2\mathrm{E}_t\left(C_{t+2}^{-1}\tilde{Y}_{t+2|t}MC_{t+2}\right)+\cdots\end{array}\right\}
\end{aligned}
$$

- Rearanging this yields:

$$
\tilde{X}_t=\mathrm{E}_t\left\{\frac{\dfrac{\varepsilon}{\varepsilon-1}\left[\begin{array}{l}C_t^{-1}\tilde{Y}_{t+k|t}MC_t+\theta\beta\left(C_{t+1}^{-1}\tilde{Y}_{t+1|t}MC_{t+1}\right)\\+(\theta\beta)^2\left(C_{t+2}^{-1}\tilde{Y}_{t+2|t}MC_{t+2}\right)+\cdots\end{array}\right]}{\left[\begin{array}{l}C_t^{-1}\tilde{Y}_{t|t}\Pi_t^{-1}\\+\theta\beta\left(C_{t+1}^{-1}\tilde{Y}_{t+1|t}\Pi_{t+1}^{-1}\Pi_t^{-1}\right)\\+(\theta\beta)^2\left(C_{t+2}^{-1}\tilde{Y}_{t+2|t}\Pi_{t+2}^{-1}\Pi_{t+1}^{-1}\Pi_t^{-1}\right)+\cdots\end{array}\right]}\right\}
$$

- Further:

$$
\tilde{X}_t=\frac{\varepsilon}{\varepsilon-1}\mathrm{E}_t\left\{\left[\begin{array}{l}C_t^{-1}\tilde{Y}_{t+k|t}MC_t\\+\theta\beta\left(C_{t+1}^{-1}\tilde{Y}_{t+1|t}MC_{t+1}\right)\\+(\theta\beta)^2\left(C_{t+2}^{-1}\tilde{Y}_{t+2|t}MC_{t+2}\right)\\+\cdots\end{array}\right]\left[\begin{array}{l}C_t^{-1}\tilde{Y}_{t|t}\Pi_t^{-1}\\+\theta\beta\left(C_{t+1}^{-1}\tilde{Y}_{t+1|t}\Pi_{t+1}^{-1}\Pi_t^{-1}\right)\\+(\theta\beta)^2\left(C_{t+2}^{-1}\tilde{Y}_{t+2|t}\Pi_{t+2}^{-1}\Pi_{t+1}^{-1}\Pi_t^{-1}\right)\\+\cdots\end{array}\right]^{-1}\right\}
$$

- Totally differentiating this and plugging the steady state value yields:

$$
\begin{aligned}
d\tilde{X}_t=&\frac{\varepsilon}{\varepsilon-1}\left\{\begin{array}{l}(-1)C^{-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\\+(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}C^{-1}\end{array}\right\}dC_t\\
&+\frac{\varepsilon}{\varepsilon-1}\left\{\begin{array}{l}C^{-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\\-(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}C^{-1}\end{array}\right\}dY_{t|t}\\
&+\frac{\varepsilon}{\varepsilon-1}\left\{\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\right\}dMC_t
\end{aligned}
$$

$$+\frac{\varepsilon}{\varepsilon-1}\left\{\begin{array}{l}\theta\beta(-1)C^{-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\\+(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\theta\beta C^{-1}\end{array}\right\}d\mathrm{E}_t\left(C_{t+1}\right)$$

$$+\frac{\varepsilon}{\varepsilon-1}\left\{\begin{array}{l}\theta\beta C^{-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\\-(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\theta\beta C^{-1}\end{array}\right\}d\mathrm{E}_t\left(Y_{t|t+1}\right)$$

$$+\frac{\varepsilon}{\varepsilon-1}\theta\beta\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}d\mathrm{E}_t\left(MC_{t+1}\right)$$

$$+\frac{\varepsilon}{\varepsilon-1}\left\{\begin{array}{l}(\theta\beta)^2(-1)C^{-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\\+(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}(\theta\beta)^2 C^{-1}\end{array}\right\}d\mathrm{E}_t\left(C_{t+2}\right)$$

$$+\frac{\varepsilon}{\varepsilon-1}\left\{\begin{array}{l}(\theta\beta)^2 C^{-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\\-(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}(\theta\beta)^2 C^{-1}\end{array}\right\}d\mathrm{E}_t\left(Y_{t|t+2}\right)$$

$$+\frac{\varepsilon}{\varepsilon-1}(\theta\beta)^2\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}d\mathrm{E}_t\left(MC_{t+2}\right)$$

$$+\frac{\varepsilon}{\varepsilon-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\left\{\begin{array}{l}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]d\Pi_t\\+\left[\theta\beta+(\theta\beta)^2+(\theta\beta)^3\cdots\right]\mathrm{E}_t\left(\Pi_{t+1}\right)\\+\left[(\theta\beta)^2+(\theta\beta)^3+(\theta\beta)^4\cdots\right]\mathrm{E}_t\left(\Pi_{t+1}\right)\\+\cdots\end{array}\right\}$$

- Further:

$$d\tilde{X}_t=\frac{\varepsilon}{\varepsilon-1}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\left[\begin{array}{l}dMC_t+\theta\beta d\mathrm{E}_t\left(MC_{t+1}\right)\\+(\theta\beta)^2 d\mathrm{E}_t\left(MC_{t+2}\right)+\cdots\end{array}\right]$$

$$+\frac{\varepsilon}{\varepsilon-1}(MC)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\left\{\begin{array}{l}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]d\Pi_t\\+\theta\beta\left[\begin{array}{l}1+\theta\beta+(\theta\beta)^2\\+\cdots\end{array}\right]d\mathrm{E}_t\left(\Pi_{t+1}\right)\\+(\theta\beta)^2\left[\begin{array}{l}1+\theta\beta+\\(\theta\beta)^2+\cdots\end{array}\right]d\mathrm{E}_t\left(\Pi_{t+2}\right)\end{array}\right\}$$

- Then:

$$d\tilde{X}_t=\frac{\varepsilon}{\varepsilon-1}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\left[\begin{array}{l}dMC_t+\theta\beta d\mathrm{E}_t\left(MC_{t+1}\right)\\+(\theta\beta)^2 d\mathrm{E}_t\left(MC_{t+2}\right)+\cdots\end{array}\right]$$

$$+\frac{\varepsilon}{\varepsilon-1}(MC)\left\{d\Pi_t+\theta\beta d\mathrm{E}_t\left(\Pi_{t+1}\right)+(\theta\beta)^2 d\mathrm{E}_t\left(\Pi_{t+2}\right)+\cdots\right\}$$

- Now we think on:

$$1+\theta\beta+(\theta\beta)^2+\cdots \qquad (14.21)$$

- Multiplying $\theta\beta$ on the both sides of Eq.(14.21) yields:

$$J\equiv 1+\theta\beta+(\theta\beta)^2+\cdots$$

- Subtracting this from Eq.(14.21) yields:

$$\theta\beta J=\theta\beta+(\theta\beta)^2+(\theta\beta)^3+\cdots$$

- Because J is defined by Eq.(14.21) and we have $J(1-\theta\beta)=1$
  Then:

$$1+\theta\beta+(\theta\beta)^2+\cdots=\frac{1}{1-\theta\beta} \qquad (14.22)$$

- Eq.(14.17) implies that following is applied in the steady state:

$$MC = \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{-1} \qquad (14.23)$$

- Eq.(14.23) implies that the real marginal cost equals to the inverse of constant markup in the steady state.
- By using Eqs.(14.22) and (14.23) ,we can be rewritten Eq.(14.17), log-linearized FONC for firms as follows:

$$\tilde{p}_t - p_{t-1} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(mc_{t+k}\right) + \sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(\pi_{t+k}\right)$$

where we use $v_t \equiv dV_t/V$ and $\tilde{x}_t = \tilde{p}_t - p_{t-1}$. This is obviously equal to Eq.(14.18).

QED

Proof of Eq.(14.19)

- Here, we thin the second term on the RHS in Eq.(14.18). The second term on the RHS in Eq.(14.18) can be rewritten as:

$$\begin{aligned}
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(\pi_{t+k}\right) &= \pi_t + \theta\beta\mathrm{E}_t\left(\pi_{t+1}\right) + (\theta\beta)^2 \mathrm{E}_t\left(\pi_{t+1}\right) + \cdots \\
&= p_t - p_{t-1} + \theta\beta\mathrm{E}_t\left(p_{t+1}\right) - \theta\beta p_t \\
&\quad + (\theta\beta)^2 \mathrm{E}_t\left(p_{t+2}\right) - (\theta\beta)^2 \mathrm{E}_t\left(p_{t+1}\right) + \cdots \\
&= -p_{t-1} + (1-\theta\beta)p_t + \theta\beta(1-\theta\beta)\mathrm{E}_t\left(p_{t+1}\right) \\
&\quad + (\theta\beta)^2(1-\theta\beta)\mathrm{E}_t\left(p_{t+2}\right) + \cdots \\
&= -p_{t-1} + (1-\theta\beta)\begin{bmatrix} p_t + \theta\beta\mathrm{E}_t\left(p_{t+1}\right) \\ + (\theta\beta)^2 \mathrm{E}_t\left(p_{t+2}\right) + \cdots \end{bmatrix}
\end{aligned}$$

- Then, we get:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(\pi_{t+k}\right) = -p_{t-1} + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(p_{t+k}\right)$$

- Plugging this into Eq.(14.18) yields:

$$\begin{aligned}
\tilde{p}_t &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(mc_{t+k} + p_{t+k}\right) \\
&= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(mc^n_{t+k}\right)
\end{aligned}$$

which equals to Eq.(14.19) obviously.

QED

- (14.18)can be rewritten as differential equation as follows:

$$\tilde{p}_t - p_{t-1} = \theta\beta\mathrm{E}_t\left(\tilde{p}_{t+1} - p_t\right) + (1-\theta\beta)mc_t + \pi_t \qquad (14.24)$$

- Plugging Eq. (14.15) into Eq.(14.24) yields:

$$\pi_t = \beta\mathrm{E}_t\left(\pi_{t+1}\right) + \kappa mc_t \qquad (14.25)$$

where $\kappa \equiv (1-\vartheta)(1-\vartheta\beta)/\vartheta$.

- κ is the slope of the NKPC. The higher the price stickiness ϑ, the lower the slope and vice versa.

- Solving Eq.(14.25) forward yields:

$$\pi_t = \kappa\sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(mc_{t+k}\right) \qquad (14.26)$$

- Eq.(14.26) shows that the inflation is netpresent value of percentage deviation of marginal cost from its steady state value.
- When the marginal cost exceeds its steady state value, namely, the inverse of constant markup $\varepsilon/(\varepsilon-1)$, inflation rises and vice versa.

Proof of Eq.(14.24)

- Rewrite Eq.(14.18) as follows:

$$\begin{aligned}
\tilde{p}_t - p_{t-1} &= (1-\theta\beta)\mathrm{E}_t\left[mc_t + \theta\beta mc_{t+1} + (\theta\beta)^2 mc_{t+2} + \cdots\right] \\
&\quad + \mathrm{E}_t\left[\pi_t + \theta\beta\pi_{t+1} + (\theta\beta)^2 \pi_{t+2} + \cdots\right] \\
&= (1-\theta\beta)mc_t + \pi_t + (1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(mc_{t+k}\right) \qquad (14.26) \\
&\quad + \sum_{k=1}^{\infty}(\theta\beta)^k \mathrm{E}_t\left(\pi_{t+k}\right)
\end{aligned}$$

- Forwarding this one period yields:

$$\mathrm{E}_t(\tilde{p}_{t+1}-p_t)=(1-\theta\beta)\mathrm{E}_t\left[mc_{t+1}+\theta\beta mc_{t+2}+(\theta\beta)^2 mc_{t+3}+\cdots\right]$$
$$+\mathrm{E}_t\left[\pi_{t+1}+\theta\beta\pi_{t+2}+(\theta\beta)^2\pi_{t+3}+\cdots\right]$$
$$=\frac{1-\theta\beta}{\theta\beta}\mathrm{E}_t\left[\theta\beta mc_{t+1}+(\theta\beta)^2 mc_{t+2}+(\theta\beta)^3 mc_{t+3}+\cdots\right]$$
$$+\frac{1}{\theta\beta}\mathrm{E}_t\left[\theta\beta\pi_{t+1}+(\theta\beta)^2\pi_{t+2}+(\theta\beta)^3\pi_{t+3}+\cdots\right]$$
$$=\frac{1-\theta\beta}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k\mathrm{E}_t(mc_{t+k})+\frac{1}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k\mathrm{E}_t(\pi_{t+k})$$

- Multiplying $\vartheta\beta$ on the both sides of this yields:

$$\vartheta\beta\mathrm{E}_t(\tilde{p}_{t+1}-p_t)=(1-\vartheta\beta)\sum_{k=1}^{\infty}(\vartheta\beta)^k\mathrm{E}_t(mc_{t+k})+\sum_{k=1}^{\infty}(\vartheta\beta)^k\mathrm{E}_t(\pi_{t+k})$$

- Plugging this into Eq.(14.26) yields:

$$\tilde{p}_t-p_{t-1}=(1-\theta\beta)mc_t+\pi_t+\theta\beta\mathrm{E}_t(\tilde{p}_{t+1}-p_t)$$

which is Eq.(14.24) itself.

QED

Proof of Eq.(14.25)

- Plugging Eq. (14.15) into Eq.(14.24) yields:

$$\frac{1}{1-\theta}\pi_t=\pi_t+(1-\theta\beta)mc_t+\frac{\theta\beta}{1-\theta}\mathrm{E}_t(\pi_{t+1})$$

which can be rewritten as:

$$\pi_t=\beta\mathrm{E}_t(\pi_{t+1})+\kappa mc_t$$

- This is Eq.(14.25) itself.

QED

Proof of (14.26)

- Rewrite Eq.(14.26) as follows:

$$\pi_t=\kappa\sum_{k=0}^{\infty}\beta^k\mathrm{E}_t(mc_{t+k})$$
$$=\kappa mc_t+\kappa\beta\mathrm{E}_t(mc_{t+1})+\kappa\beta^2\mathrm{E}_t(mc_{t+2})+\cdots$$
$$=\kappa mc_t+\kappa\sum_{k=1}^{\infty}\beta^k\mathrm{E}_t(mc_{t+k}) \qquad (14.27)$$

- By leading Eq.(14.27) one period and multiplying β yields:

$$\beta\mathrm{E}_t(\pi_{t+1})=\kappa\beta\mathrm{E}_t(mc_{t+1})+\kappa\beta^2\mathrm{E}_t(mc_{t+2})+\kappa\beta^3\mathrm{E}_t(mc_{t+3})+\cdots$$
$$=\kappa\sum_{k=1}^{\infty}\mathrm{E}_t(mc_{t+k})$$

- Plugging this into Eq.(14.27) yields:

$$\pi_t=\beta\mathrm{E}_t(\pi_{t+1})+\kappa mc_t$$

which is Eq.(14.25) itself. Thus, Eq. (14.26) can be derived from Eq. (14.25).

QED